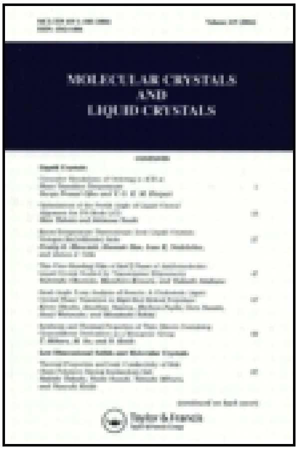

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Time Dispersion, Waves Irreversibility and Absorption Effects in Cholesteric Liquid Crystals

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Time Dispersion, Waves Irreversibility and Absorption Effects in Cholesteric Liquid Crystals

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The interaction of light with Cholesteric Liquid Crystals (CLC) in the presence of time dispersion, magnetooptical activity and absorption is considered. The magnetooptical activity leads to the waves irreversibility, i.e. to the absence of center of symmetry for the surface of wave vectors and other surfaces, characterizing the direction dependence of optical properties (the nonequivalency of the forward and backward directions of propagation occurs). The waves irreversibility effects in CLC and their influence on the geometry of the Bragg reflection are considered.

The time dispersion leads both to the splitting of region of diffraction reflection, and to the appearance of new region of diffraction reflection. Near the isotropic point it is possible to get a narrow region of diffraction reflection with tunable width and location.

The results present also interest for the electrodynamics of media with waves irreversibility. They can be used no only in CLC optics but also in microwave electrodynamics of artificial ferromagnetic helical media.

Keywords: liquid crystals; helical structures; wave irreversibility

INTRODUCTION

Here we shall consider a number of features in optical properties of dispersing cholesteric liquid crystals (CLC). Results obtained thereat will mainly be based on the analysis of the dispersion equation. The latter, as it is generally known, even in the absence of a time (frequency) dispersion of dielectric permittivity is not a simple function between wave vector and frequency. In part 1 of this paper we shall consider the features of the dispersion equation in the presence of the time dispersion of the dielectric permittivity. As it will be shown, the time dispersion, being the cause of considerable change of the wave vector-frequency

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dependence, leads to the splitting of the region of diffraction reflection (RDR) and to the appearance of new RDR in the vicinity of absorption frequencies.

Interesting properties are manifested in the presence of the isotropic point, the frequency ω_{is}, at which the curves of frequency dependence of ε_1 and ε_2 intersect (here and everywhere ε_1 and ε_2 are principal values of the tensor of dielectric permittivity in the planes perpendicular to the medium axis). In such a situation it appears to be possible to get the small- width RDR. With this there is possibility to govern the width and location of RDR on the frequency axis.

In the presence of time dispersion and external magnetic field, as it is known[1], the magneto-optical activity arises in any medium. If, moreover, the spatial structure of the medium possesses an left hand-right hand asymmetry, then the waves irreversibility in the medium (violation of optical reversibility : absence of a center of symmetry of wave vectors surface and resulting absence of a center of symmetry of other surfaces describing the dependence of optical characteristics on direction of propagation) appears. Irreversibility of waves for homogeneous gyrotropic media originally was investigated by the author (see part 2). Such media possess, as is known[2], an mentioned above asymmetry of the spatial structure. Because the CLC also possess such an asymmetry[3], the wave irreversibility must appear in them.

The waves irreversibility effects in CLC are considered in the part 2 of the present paper.

The wave irreversibility leads also to the new features in the geometry of Bragg reflection. These features are considered in the last item of the part 2.

The dispersion near the resonance frequencies is accompanied by absorption. The absorption effects are considered in part 3. The irreversibility of circular dichroism (stipulated by absorption), the polarization selectivity of CLC near isotropic point and the mentioned above selectivity at the isotropic point in the presence of absorption anisotropy are considered.

Everywhere below we shall assume that the axis of CLC coincides with the axis z of the laboratory system x, y, z. The propagation direction of waves in CLC also coincides with axis z.

1. EFFECTS OF DISPERSION

1.1. Dispersion equation in a local system. Geometrical interpretation of diffraction reflection region appearance in the absence of time dispersion

Let the electromagnetic wave of frequency ω

$$\vec{E}(z,t) = \vec{E}(z)\exp(-i\omega t) \quad (1)$$

is propagating in a CLC. The principal directions of the dielectric tensor ε_{ij} in the plane $z = 0$ coincide with the x and y axes. The values of tensor ε_{ij} along those axes are equal to ε_1 and ε_2 respectively. Following[4] let introduce the system of axes x', y', z. The axes x', y' turn together with the structure and in any plane

z =const coincide with the principal directions of ε_{ij}. The system x', y', z, as is known, is named "local".

Let represent the field in a medium in the form:

$$\vec{E}(z,t) = \sum_m \{[(E_{mx}\cos az + E_{my}\sin az)\vec{x}^0 + \\ +(E_{my}\cos az - E_{mx}\sin az)\vec{y}^0]\exp i(K_m z - \omega t)\}. \quad (2)$$

Here $a = \dfrac{2\pi}{\sigma}$, σ is the pitch of helix, E_{mx}, E_{my} are the amplitudes of the field components along axes x', y', respectively. These components vary in space with period $2\pi / K_m$ (in x', y', z axes system). $\vec{x}^0$, $\vec{y}^0$ are unit vectors of x and y axes.

Making a transition to the components related to x' and y' axes, we can also represent the magnetic field of the wave in the form analogous to (2). Substituting the obtained by such way expression of the magnetic field and expression (2) into Maxwell's equations, we come to the following equation in the local system (see, for example[5–7]):

$$\left(\frac{\omega^2}{c^2}\varepsilon_1 - K_m^2 - a^2\right)\left(\frac{\omega^2}{c^2}\varepsilon_2 - K_m^2 - a^2\right) - 4a^2K_m^2 = 0 \quad (3)$$

or

$$K_m^4 - \left(\frac{\omega^2}{c^2}\varepsilon_1 + \frac{\omega^2}{c^2}\varepsilon_2 + 2a^2\right)K_m^2 + \left(\frac{\omega^2}{c^2}\varepsilon_1 - a^2\right)\left(\frac{\omega^2}{c^2}\varepsilon_2 - a^2\right) = 0. \quad (4)$$

For K_m we obtain:

$$K_m^2 = \frac{\omega^2}{c^2}\frac{\varepsilon_1 + \varepsilon_2}{2} + a^2 \pm \sqrt{\left(\frac{\omega^2}{c^2}\frac{\varepsilon_1 - \varepsilon_2}{2}\right)^2 + 4a^2\frac{\omega^2}{c^2}\frac{\varepsilon_1 + \varepsilon_2}{2}}. \quad (5)$$

Below, the upper sign before the radical in (5), will correspond to the indices m=1,2, the lower sign therein-to the indices m=3,4:

$$K_{1,2} = \pm\sqrt{K_{1,2}^2}, \quad K_{3,4} = \pm\sqrt{K_{3,4}^2}.$$

The region of diffraction reflection is the region in which $K_{3,4}^2 < 0$. [3,4] The boundaries of RDR are defined as the roots of $K_{3,4}^2 = 0$ equation. The latter leads to the following relations:

$$\frac{\omega^2}{c^2}\varepsilon_1 - a^2 = 0, \quad \frac{\omega^2}{c^2}\varepsilon_2 - a^2 = 0. \tag{6}$$

From (6) we find that the frequency boundaries ω_1, ω_2 of RDR in the absence of ε_1 and ε_2 dispersion:

$$\omega_1 = \frac{ac}{\sqrt{\varepsilon_1}}, \quad \omega_2 = \frac{ac}{\sqrt{\varepsilon_2}}. \tag{7}$$

On the Figure 1 the geometrical illustration of the RDR appearance is given. The straight line 1 describes the function $y_1 = \frac{\omega}{c}\sqrt{\varepsilon_1}$, the straight line 2- the function $y_2 = \frac{\omega}{c}\sqrt{\varepsilon_2}$, and the straight line 3- the constant $y_3 = a$.

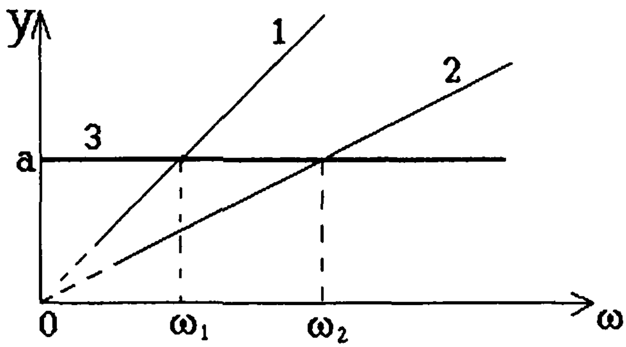

FIGURE 1

At the one boundary of RDR we have $y_3 = y_1$, at the other one- $y_3 = y_2$. Inside of this region we have $y_2 < y_3 < y_1$ (to be concrete we consider $\varepsilon_1 > \varepsilon_2$). The latter inequality follows also from (4).

Thus, RDR is the region in which the straight line $y_3 = a$ passes between the lines $y_1 = \frac{\omega}{c}\sqrt{\varepsilon_1}$ and $y_2 = \frac{\omega}{c}\sqrt{\varepsilon_2}$. Such geometrical illustration of RDR appearance we shall use in the following item.

1.2. The case of ε_{ij} components dependence on frequency

In the item 1.1 we obtained the values of boundary frequencies of RDR (ω_1 and ω_2) in absence of frequency dependence of ε_1 and ε_2. Let consider now the situation when there is frequency dispersion of those quantities. We shall consider the simple case when in some frequency region the quantities $\varepsilon_1(\omega)$ and $\varepsilon_2(\omega)$ depend on ω according to the same law:

$$\varepsilon_1(\omega) = \varepsilon_1 + \delta\varepsilon(\omega), \quad \varepsilon_2(\omega) = \varepsilon_2 + \delta\varepsilon(\omega), \tag{8}$$

$$\delta\varepsilon(\omega) = A\frac{(\omega_0^2 - \omega^2) + i\omega\gamma}{(\omega_0^2 - \omega^2)^2 + (\omega\gamma)^2}. \tag{9}$$

Here ω_0 is the frequency of the given optical transition causing the dispersion.

If $|\omega - \omega_0| >> \gamma$, then $|\delta\varepsilon''| << |\delta\varepsilon'|$. Then, neglecting $\delta\varepsilon''$ in comparison with $\delta\varepsilon'$ (this condition maintains its meaning at $A \neq 0$) and substituting (8), (9) into (6), we obtain two equations for ω. The roots of these equations are:

$$\omega_1^{\pm} = \frac{1}{\sqrt{2}}\left[\left(\omega_0^2 + \omega_1^2 + \frac{A}{\varepsilon_1}\right) \pm \sqrt{\left(\omega_0^2 + \omega_1^2 + \frac{A}{\varepsilon_1}\right)^2 - 4\omega_0^2\omega_1^2}\right]^{\frac{1}{2}}, \tag{10}$$

$$\omega_2^{\pm} = \frac{1}{\sqrt{2}}\left[\left(\omega_0^2 + \omega_2^2 + \frac{A}{\varepsilon_2}\right) \pm \sqrt{\left(\omega_0^2 + \omega_2^2 + \frac{A}{\varepsilon_2}\right)^2 - 4\omega_0^2\omega_2^2}\right]^{\frac{1}{2}}. \tag{10a}$$

Here ω_1 and ω_2 are given by the relations (7), in which ε_1 and ε_2 don't depend on ω.

In Figure 2 the approximate run of $\frac{\omega}{c}\sqrt{\varepsilon_1(\omega)}$ and $\frac{\omega}{c}\sqrt{\varepsilon_2(\omega)}$ is presented. The anomalous dispersion region, in which the condition $|\delta\varepsilon''| << |\delta\varepsilon'|$ is not fulfilled, is the region 3–4. In the regions 1–2 and 5–6 the straight line $y_3 = a$ (straight line 3, Figure 2) passes between the curves $y_1 = \frac{\omega}{c}\sqrt{\varepsilon_1(\omega)}$ and $y_2 = \frac{\omega}{c}\sqrt{\varepsilon_2(\omega)}$ (the curves 1 and 2).

Therefore $\frac{\omega^2}{c^2}\varepsilon_1(\omega) - a^2$ and $\frac{\omega^2}{c^2}\varepsilon_2(\omega) - a^2$ have the opposite signs. Hence, in this regions (1–2 and 5–6), according to (4), $K_{3,4}^2 < 0$.

Thus, it turned out that we have two RDR : 1–2 and 5–6. The four boundaries of these regions are $\omega_1^{\pm}$, $\omega_2^{\pm}$.

Our conclusions about existence of two regions of diffraction reflection are confirmed by the exact calculation of reflection coefficient R. In Figure 3 the dependence of R on ω is presented. The light with diffracting polarization is incident on a semispace, occupied by a CLC. The pitch of helix $\sigma = 0.42\mu m$,

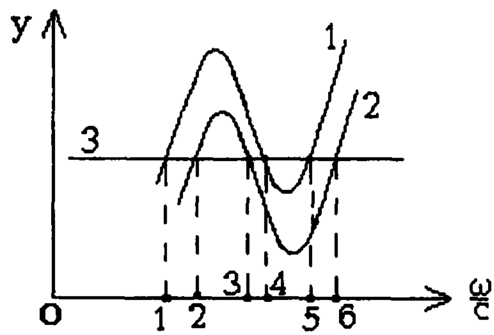

FIGURE 2

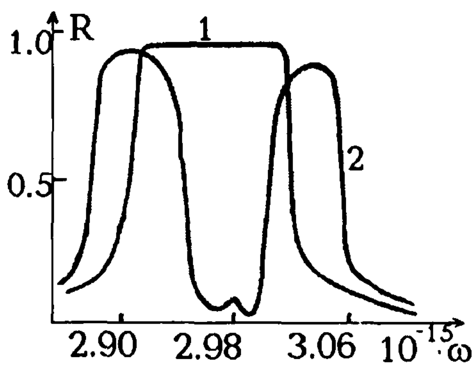

FIGURE 3

$A = 2\cdot10^{28}$ sec^{-2}, $\gamma = 4\cdot10^{12}$ sec^{-1}, $\omega_0 = 2{,}38\cdot10^{15}$ sec^{-1}, $\varepsilon_1 = 2.29$, $\varepsilon_2 = 2{,}14$. The curve 1 corresponds to the nondispersive CLC, the curve 2-dispersive. The parameters of CLC for curves 1 and 2 are the same. The difference is only in that in the first case $\delta\varepsilon(\omega) = 0$, in the second one, corresponding to thepresence of the absorption line, $\delta\varepsilon(\omega) \neq 0$. As it is seen from the curves, the time dispersion has leaded to the splitting of RDR with creation of two such regions[7,8].

Let imagine that originally we have the nondispersive CLC and then introduce to it an impurity with the absorption frequency in RDR. Then we get the described picture of RDR splitting. But if the dispersion region lies beyond RDR, then we obtain a new RDR in that region (if, of course, the line $y_3 = a$ is intersected with curves $y_1 = \frac{\omega}{c}\sqrt{\varepsilon_1(\omega)}$ and $y_2 = \frac{\omega}{c}\sqrt{\varepsilon_2(\omega)}$ as it was described before (Figure 2).

1.3. The case of isotropic point existence

The frequency dispersion may lead to the existence of the intersection point of the curves $\varepsilon_1(\omega)$ and $\varepsilon_2(\omega)$. Such situations for homogeneous anisotropic media were considered, for instance, in[9].

Let the above mentioned intersection point is named the isotropic point[9]. Below we shall consider the CLC having the isotropic point.

Consider the narrow frequency region in which the curves of frequency dependence of ε_1 and ε_2 are intersected at some frequency $\omega = \omega_{is}$. We shall consider that ω_{is} is in the region of normal dispersion of $\varepsilon_1(\omega)$ and $\varepsilon_2(\omega)$, and the absorption can be neglected.

In Figure 4 the dependence $y_1 = \frac{\omega}{c}\sqrt{\varepsilon_1(\omega)}$ and $y_2 = \frac{\omega}{c}\sqrt{\varepsilon_2(\omega)}$ near $\omega = \omega_{is}$ are presented. Let denote the values of $\varepsilon_1(\omega)$ and $\varepsilon_2(\omega)$ at $\omega = \omega_{is}$ by ε_{is}. If the helix pitch has the value satisfying the condition

$$\frac{\omega_{is}}{c}\sqrt{\varepsilon(\omega_{is})} = a_0, \tag{11}$$

then the existence of RDR is not possible) $a_0 = \frac{2\pi}{\sigma}$, σ -the pitch of helix).

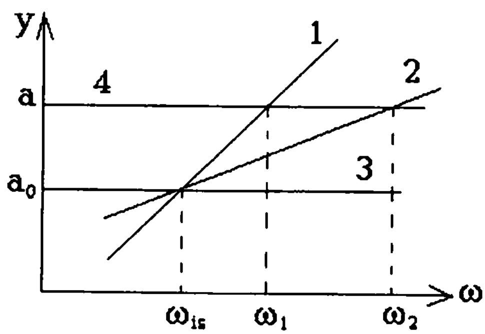

FIGURE 4

Really, the quantities $\frac{\omega^2}{c^2}\varepsilon_1(\omega) - a^2$ and $\frac{\omega^2}{c^2}\varepsilon_2(\omega) - a^2$ near ω_{is} have the same signs (see Figure 4), meanwhile for the existence of RDR they have to have opposite signs (see item 1.2). The straight lines 1,2,3 in Figure 4 describe the dependence $y_1 = \frac{\omega}{c}\sqrt{\varepsilon_1(\omega)}$, $y_2 = \frac{\omega}{c}\sqrt{\varepsilon_2(\omega)}$, $y_3 = a_0$ near the isotropic point. With the variation of the helix pitch the quantity $\frac{2\pi}{\sigma}$ will be changed by some quantity Δa. Then the straight line $y_4 = a_0 + \Delta a$ will intersect the lines 1 and 2 at different points, but not at the isotropic point. There appears the RDR with boundary frequencies ω_1 and ω_2.

In Figure 5 depicted are the curves of dependence of the reflection coefficient on wavelength (the wavelength relates to vacuum). The light with diffracting circular polarization incidences normally on a half-space occupied by a CLC.

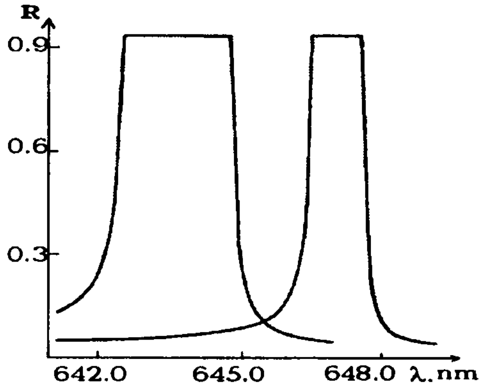

FIGURE 5

The left curve corresponds to the case $a = a_0 - 0{,}02a_0$, the right one-to the case $a = a_0 - 0{,}01a_0$. $\varepsilon_{is} = 2{,}2165$, $\lambda_{is} = \dfrac{2\pi c}{\omega_{is}} = 650nm$, $\varepsilon_1'' = \varepsilon_2'' = 10^{-7}$. $\varepsilon_{1,2}' = \varepsilon_{is} + b_{1,2}(\lambda - \lambda_{is})$, $b_1 = -10\mu m^{-1}$, $b_2 = -5\mu m^{-1}$, $a_0 = \dfrac{\omega_{is}}{c}\sqrt{\varepsilon_{is}}$.

As it is seen from the figure, a narrow RDR is formed, the position and the width of which depend on Δa. So, at $\Delta a = 0.02a_0$ RDR width is about 2.5*nm*, and at $\Delta a = 0.01a_0$-about 1.5*nm* (25 Å and 15 Å respectively).

Thus, the CLC with an isotropic point enable to get narrow RDR with adjustable width and position. Such changes of helix pitch one can achieve by external influences, for instance, by applying magnetic or electric fields[6,10].

2. IRREVERSIBILITY OF WAVES

In a number of works (see, for instance[11,12,13]), summarized in[7,14], the interaction of an electromagnetic wave with naturally gyrotropic media in the presence of a magnetooptical activity has been studied. These studies have shown that in such media the surface of wave vectors is deprived of the center of symmetry. That is, phase velocities of waves propagating in mutually opposite directions (not perpendicular to the direction of the external magnetic field) differ from each other by modules.

Thus, in the simplest case of the isotropic naturally gyrotropic medium in the presence of external magnetic field $\vec{H}$ we have[14]:

$$k^{\pm 2} = \frac{\omega^2}{c^2}[\varepsilon \pm (\gamma + g\cos\alpha)], \quad \cos\alpha = \frac{\vec{k}^{\pm}\vec{H}}{|\vec{k}^{\pm}||\vec{H}|}. \tag{12}$$

There ε is the dielectric permittivity, γ describes the natural activity and g -the magnetooptical activity, $\vec{k}^{\pm}$ are the wave vectors of waves with right and left circular polarization, α is the angle between direction of propagation and $\vec{H}$. For the backward direction of propagation we obtain (changing $\cos\alpha \to -\cos\alpha$ in (12)):

$$k_{back}^{\pm 2} = \frac{\omega^2}{c^2}[\varepsilon \pm (\gamma - g\cos\alpha)]. \tag{13}$$

As it follows from (12) and (13), the surface describing the dependence of $\vec{k}^{\pm}$ on α (the surface of wave vectors) is deprived of the center of symmetry:

$$k_{back}^{+} \neq k_{forw}^{+},\ k_{back}^{+} \neq k_{forw}^{-} \quad k_{back}^{-} \neq k_{forw}^{-}\ k_{back}^{-} \neq k_{forw}^{+}$$

i.e. the waves irreversibility occurs.

The wave irreversibility leads also to the absence of the center of symmetry for the other surfaces describing dependence of other quantities (for instance, of polarization plane rotation, circular dichroism and so on) on direction of propagation.

Note that irreversibility of the waves leads also to the change of geometry of reflection and refraction of rays. The well known in optics a reversibility of light rays is violated. The reason for such irreversibility is not in the Maxwell's equations, it is in the material equations. It is connected, as it mentioned above, with the simultaneous presence of a magnetooptical activity and right hand-left hand asymmetry of the space structure of a medium.

The subsequent studies have shown that the irreversibility of waves takes place also in CLC in the presence of a magnetic field directed along the axis of medium[7,15,16].

The appearance of irreversibility in a CLC must be explained in the same way as its appearance in homogeneous gyrotropic media[14]. Let consider the simple case of wave propagation along the axis of a CLC. Then, if in case of a given direction of wave propagation along the axis of a CLC two rotations of the polarization plane (one of them is caused by the twisting of a medium, the other one-by the presence of a magnetooptical activity) are added, then in case of reverse direction of propagation they are subtracted (since the direction of the magnetooptical rotation changes its direction when the propagation direction is changed to the reverse one). And this means that two mutually opposite directions (forward and backward directions of propagation) are not equivalent. In the dispersion equation, as it will be seen, this non-equivalency is manifested by the presence of the odd power components of the wave vector, such in the natural gyrotropic media in presence of the magnetic field[14] (see (12)).

2.1 Dispersion equation in a local system in the presence of a magnetic field

The dispersion equation for a CLC being in the external magnetic field aligned with the helix axis has the form:

$$\left(\frac{\omega^2}{c^2}\varepsilon_1 - K_{mg}^2 - a^2\right)\left(\frac{\omega^2}{c^2}\varepsilon_2 - K_{mg}^2 - a^2\right) - \left(2aK_{mg} + \frac{\omega^2}{c^2}g\right)^2 = 0. \quad (14)$$

The parameter g is responsible for the magnetooptical activity induced by the magnetic field.

The quantities ε_1 and ε_2 are the values of the diagonal components of the dielectric tensor (in the local system x', y', z.) established in the magnetic field. The index "g" indicates the presence of the magnetooptical activity.

The equation (14) differs from (3) and, in general, from common met dispersion equations by the presence of the odd power of K_{mg}. The odd power of K_{mg} exists in the term $4a\frac{\omega^2}{c^2}gK_{mg}$. The latter appears when we raise to the second power the quantity $\left(2aK_{mg} + \frac{\omega^2}{c^2}g\right)$ in (14). The presence of the odd power of K_{mg}means that if K_{mg} is the root of equation (14), then K_{mg} cannot be its root. Hence, all the four roots of K_{mg} ($m = 1, 2, 3, 4$) differ from each other by module. The z- components of wave vectors in the system of axis x, y, z are equal to $K_{mg}\pm a$. These quantities also differ from each other by the module. And this means that when the propagation direction is changed by the reverse one, there does not take place the coincidence of the modules of z- components of wave vectors for forward and backward waves. In other words, there takes place the waves irreversibility. As it could be expected, the irreversibility will not take place in the absence of right hand-left hand asymmetry of medium structure ($a = 0$) or in the absence of the magnetooptical activity ($g = 0$). In the both cases the term $4a\frac{\omega^2}{c^2}gK_{mg}$ containing the odd power of K_{mg}, vanishes.

2.2 Rotation of polarization plane under the conditions of waves irreversibility

Far from the boundaries of the RDR when $|K_{3,4g}| \gg \left(\frac{\omega}{c}g\right)$ (the quantity $\left(\frac{\omega}{c}g\right)$ in optical region for diamagnetic liquids in the fields near 10^3 Oe has on order of 10^{-2}; see, for example[11]), the roots of equation (14) are of the form (when $a \neq 0$, $|K_{mg} - K_m| << |K_m|$):

$$K_{1,2g} = K_{1,2} + \frac{\omega^2}{c^2} agB^{-1}, \tag{15}$$

$$K_{3,4g} = K_{3,4} - \frac{\omega^2}{c^2} agB^{-1}. \tag{15a}$$

Here K_m are given by the formulae (5), and

$$B = \sqrt{\left(\frac{\omega^2}{c^2}\frac{\varepsilon_1 - \varepsilon_2}{2}\right)^2 + 4a^2\frac{\omega^2}{c^2}\frac{\varepsilon_1 + \varepsilon_2}{2}}. \tag{16}$$

In homogeneous crystals, as it is well known[17], the magnetic field shifts the branches of dispersion equation related to the waves with right and left circular (elliptic) polarization to the opposite sides. The shifts take place along the straight line parallel to the direction of the external magnetic field. According to (15), (15a) the same situation is realized in CLC. Indeed, the presence of the magnetic field led to change of $K_{1,2}$ (which correspond to the waves with a non-diffracting circular polarization) by $\omega^2 agc^{-2}B^{-1}$, and $K_{3,4}$ (corresponding to the waves with diffracting polarization)-by $-\omega^2 agc^{-2}B^{-1}$.

Let consider now a rotation of the polarization plane. The rotation on the unit length of the ray path far from the boundaries of the RDR is determined by the formula[5]:

$$\vartheta = (K_1 - a) - (K_3 + a). \tag{17}$$

In the case of magnetooptical activity we shall have:

$$\vartheta_g = (K_{1g} - a) - (K_{3g} + a). \tag{18}$$

Substituting $K_{1,3g}$ from (15), (15a) into (17) being just correct far from RDR, we get

$$\vartheta_g = \vartheta + 2\frac{\omega^2}{c^2} agB^{-1}. \tag{19}$$

Then changing a direction of waves propagation to the reverse one (it is equivalent to the replacement $g \to -g$) we obtain for the rotation ($\vartheta_{-g} = \vartheta_{backw}$):

$$\vartheta_{-g} = \vartheta - 2\frac{\omega^2}{c^2} agB^{-1}. \tag{20}$$

According to (19) and (20) the rotations, corresponding to light propagation in the mutually opposite directions, are not identical by absolute value: $|\vartheta_g| \neq |\vartheta_{-g}|$. They differ in

$$\Delta\vartheta = |\vartheta_g| - |\vartheta_{-g}| = 4\frac{\omega^2}{c^2}agB^{-1} \tag{21}$$

or

$$|\vartheta_{forw}| - |\vartheta_{backw}| = 4\frac{\omega^2}{c^2}agB^{-1}. \tag{22}$$

For CLC layers with 10 μm thickness the difference $\Delta\vartheta$ corresponds approximately to 0.001 degrees (if taking again that $\frac{\omega}{c}g \sim 10^{-2}$). Such a rotations being measurable, for practical applications are small, however. We shall return to the practical aspects of this problem in Discussion. Note, once more, that both a difference of $\Delta\vartheta$ from zero and a presence of the odd power of K_{mg} in the equation (14) are caused by simultaneous presence of twisting and magnetooptical activity ($a \neq 0$, $g \neq 0$).

2.3. Light transmission coefficient through a CLC layer in the presence of waves irreversibility

In Figure 6 presented is the dependence of ΔT on the wave length. $\Delta T = T_{forw} - T_{backw}$, where T_{forw} is the transmission coefficient through a planar layer of CLC. The light passes along the direction of applied magnetic field parallel to the helix axis. T_{backw} is the same coefficient for the light transmission in the reverse direction. In both cases the light is plane-polarized. The CLC layer with parameters $\varepsilon_1 = 2.29$, $\varepsilon_2 = 2.14$, $g = 10^{-1}$, $\sigma = 0.42\mu m$ and the thickness of 200μm is placed between two plane-parallel glass plates. The thickness of each plates are 1000μm. The refraction index of plates is 1.5.

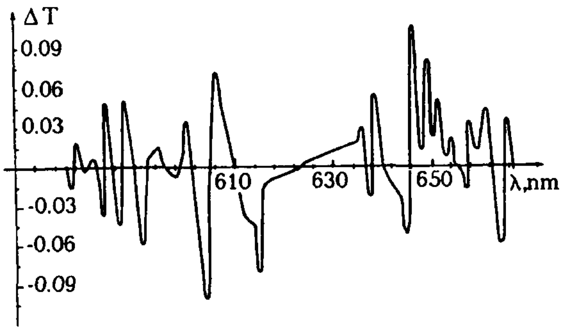

FIGURE 6

If one is guided by the maximum possible value of $T \approx 1$, then, as it is seen from Figure 6, the relative change of the transmission coefficient can reach the values of the order of 10%. It means that the transmission coefficients when light normally passes in direct and reverse directions can differ by 20%. A taken value of $g \sim 10^{-1}$ is, of course, enormous in comparison with generally possible values of g for dielectric media, magnetooptical activity of which has a diamagnetic origin. The one of possible ways of increasing g is to approach the absorption line. About other possibility to increase the irreversibility effects see Discussion.

2.4. Bragg condition in the presence of irreversibility of waves

Let consider the ordinary case of light propagation along the medium axis, using the equation (14). For the frequency boundaries ω_{1g}, ω_{2g} of RDR we can have the same expression as in (7), if the quantities of the order $\left(\frac{g}{\varepsilon_{1,2}}\right)^2$ are ignored[18]:

$$\omega_{1g} = \omega_1, \quad \omega_{2g} = \omega_2. \tag{23}$$

The values of these frequencies at $g \neq 0$ are defined from the requirement that the quantities $K_{3,4\,g}$ must be multiple roots of the equation (14) on the boundaries of RDR (see[7],[18]), therefore just the image roots of this equation with real coefficients are the multiple roots. (In the case $g = 0$, accordingly, the equation (3) on the boundaries of RDR has the multiple roots. But these roots are equal to zero, while the multiple roots of the equation (14) are not equal to zero). Substituting $\omega_{1g}$and ω_{2g} into (14), we get the values of the multiple roots. Then we can easy get the waves lengths in the situation of the Bragg reflection:

$$\lambda_{1i,s} = \sigma\left(1 \pm \frac{2g}{3\varepsilon_1 + \varepsilon_2}\right), \quad \lambda_{2i,s} = \sigma\left(1 \pm \frac{2g}{3\varepsilon_2 + \varepsilon_1}\right). \tag{24}$$

By the indices i, s the incident (direct) and scattered (reverse) waves respectively are marked. Mark 1 corresponds to the frequency ω_{1g}, mark 2-to the frequency ω_{2g}. σ -is the helix pitch. As a result of difference in the phase velocities of the direct and backward waves (because of irreversibility), the lengths of these waves are unequal. The corresponding wave numbers $(2\pi / \lambda)$ are also unequal.

In Figure 7 presented are the vector diagrams expressing the Laue equation

$$\vec{k}_s - \vec{k}_i = \vec{\tau}. \tag{25}$$

Here $\vec{k}_i$, $\vec{k}_s$ are, respectively, the wave vectors of incident and scattered waves $\left(k = \frac{2\pi}{\lambda}\right)$, $\vec{\tau}$ is the reciprocal lattice vector ($\tau = \frac{2\pi}{(\sigma/2)}n = \frac{2\pi}{d}n$, n-is

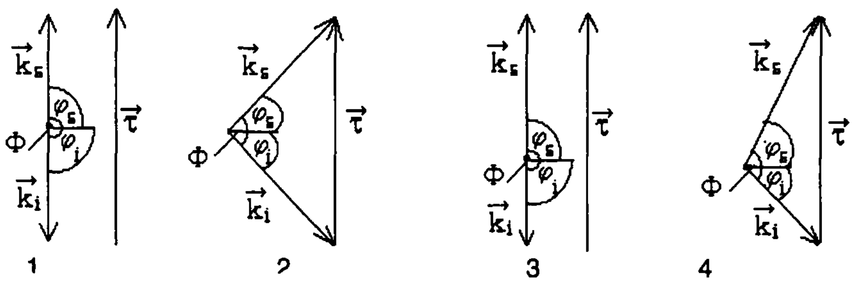

FIGURE 7

an integer, d -is the period of spacial nonhomogenity of medium). The cases 1,2 in Figure 7 correspond to the situation, when $k_i = k_s$ (ordinary cases), the cases 3,4-to the situation, when $k_i \neq k_s$. In the case 2 we have $2 \cdot \frac{2\pi}{\lambda} \sin\varphi = \frac{2\pi}{d} n$ (where $\varphi = \varphi_i = \varphi_s = \frac{\phi}{2}$), i.e. ordinary relation $2d \sin\varphi = n\lambda$.

When $\varphi = \frac{\pi}{2}$, which corresponds to the propagation of waves along the CLC axis, we have $\lambda = 2d = \sigma$ (for simplify, we take the case n=1 which occurs only, when $\varphi = \frac{\pi}{2}$).

In the cases 3,4 we have from (25), projecting latter in the direction $\vec{\tau}$ and axis x, which is perpendicular to $\vec{\tau}$ [18a]:

$$\frac{2\pi}{\lambda_i} \sin\varphi_i + \frac{2\pi}{\lambda_s} \sin\varphi_s = \frac{2\pi}{d}, \tag{26}$$

$$\frac{2\pi}{\lambda_i} \cos\varphi_i = \frac{2\pi}{\lambda_s} \cos\varphi_s. \tag{27}$$

The equation (27) expresses the continuity of wave-vectors component parallel to layers. At $\varphi_i = \frac{\pi}{2}$ we have also $\varphi_s = \frac{\pi}{2}$ (from the second equation).

Taking $\varphi_i = \frac{\pi}{2}$ we can get for the wavelengths: $\frac{2\pi}{\lambda_{1i}} + \frac{2\pi}{\lambda_{1s}} = \frac{2\pi}{d}$ and $\frac{2\pi}{\lambda_{2i}} + \frac{2\pi}{\lambda_{2s}} = \frac{2\pi}{d}$. Note that wavelengths (24) satisfy the latter two relations (for frequencies ω_{1g} and ω_{2g} separately) to that accuracy in $\frac{g}{\varepsilon_{1,2}}$, to which those wavelengths are calculated. Note also, that substituting $\varphi_i = \frac{\pi}{2}$, in the absence of irreversibility ($\lambda_{1i} = \lambda_{1s}$, $\lambda_{2i} = \lambda_{2s}$) we obtain from (26), (27), (24) $\lambda = \sigma$. But in the presence of irreversibility λ_i and λ_i differ from σ.

3. ABSORPTION EFFECTS

The influence of absorption on the optical properties of CLC is considered in a number of works (see, for example [3]). Below we consider the situation, when the helical unhomogenity of the medium is connected only with anisotropy of absorption only [19] : $\varepsilon_1'' - \varepsilon_2'' \neq 0, \varepsilon_1' - \varepsilon_2' = 0$. Such a situations may occur, for example, at the isotropic point frequency ω_{is} when an absorption is taken into account $(\varepsilon_{1,2}'' \neq 0)$. It is interesting to consider a frequency shifts from ω_{is}: in such a situation the medium possess anisotropy, but does not posses RDR with finite width (when relation $a = \frac{\omega_{is}}{c}\sqrt{\varepsilon_{is}}$ occurs). Below such a situation is considered too.

As it is known, the imaginary parts of K_m, which stipulate the circular dichroism, may own be stipulated by diffraction reflection as well as by absorption. In the last item the irreversibility of the circular dichroism stipulated by absorption is considered.

3.1 Absorption of the waves with diffracting and non-diffracting polarization in isotropic point

Let consider the narrow frequency region including ω_{is}. Let the frequency ω of the wave differ from ω_{is} by the $\Delta\omega$. The quantities $\varepsilon_{1,2}(\omega)$ may be presented in the form :

$$\varepsilon_{1,2}(\omega) = \varepsilon_{is} + i\varepsilon_{1,2}''(\omega_{is}) + (\varepsilon_{1,2}(\omega_{is} + \Delta\omega) - \varepsilon_{1,2}(\omega_{is})). \qquad (28)$$

Here ε_{is} is the real part of $\varepsilon_{1,2}(\omega)$ at the frequency $\omega=\omega_{is}$: $\varepsilon_{is} = \varepsilon_1'(\omega_{is}) = \varepsilon_2'(\omega_{is})$. If $\varepsilon_{1,2}'' \neq 0$, at the frequency of coincidence of ε_1' and ε_2' we will have a helical medium with inhomogenity stipulated only by the absorption anisotropy.

The quantities $\frac{\omega^2}{c^2}\varepsilon_{1,2}(\omega)$ may be presented in the form:

$$\frac{\omega^2}{c^2}\varepsilon_{1,2}(\omega) = a^2(1 + x_{1,2}). \qquad (29)$$

$$x_{1,2} = 2\frac{\Delta\omega}{\omega_{is}} + \left(\frac{\Delta\omega}{\omega_{is}}\right)^2 + \\ + \left[i\varepsilon_{1,2}''(\omega_{is}) + (\varepsilon_{1,2}(\omega) - \varepsilon_{1,2}(\omega_{is}))\left(1 + \frac{\Delta\omega}{\omega_{is}}\right)^2\right](\varepsilon_{is})^{-1}. \qquad (30)$$

In latter we shall assume that $|x_{1,2}| \ll 1$. Ignoring the quantities proportional to $x_{1,2}$ of third and higher powers, we obtain (substituting (29) into (30)):

$$K_{1,2}^2 = 4a^2\left(1 + \frac{x_1 + x_2}{4} - \frac{x_1 x_2}{16}\right), \qquad (31)$$

$$K_{3,4}^2 = a^2 \frac{x_1 x_2}{4}. \tag{31a}$$

Let consider now the isotropic point ($\Delta\omega = 0$):

$$x_{1,2} = i\frac{\varepsilon''_{1,2}}{\varepsilon_{is}} \quad (\varepsilon''_{1,2} = \varepsilon''_{1,2}(\omega_{is})). \tag{32}$$

Ignoring $x_{1,2}$ in the second and higher powers, we obtain from (31), (31a) and (32):

$$\text{Im } K_{1,2} = \pm a\frac{\varepsilon''_1 + \varepsilon''_2}{4}, \quad \text{Im } K_{3,4} = \pm a\sqrt{\frac{\varepsilon''_1\varepsilon''_2}{4}}. \tag{33}$$

The polarization selective diffraction reflection in this case occurs too (see item 3.2, the relations (36)). According to (33), we have

$$|\text{Im}K_{3,4}| \leq |\text{Im}K_{1,2}|, \tag{34}$$

i.e. when the nonhomogenity is caused by absorption anisotropy only the diffracting wave has smaller absorption than the non-diffracting wave.

3.2 Boundary problem

Let a plane wave incidences normally on a boundary $z = 0$ of CLC, which occupies the region $z \geq 0$. The following relations between the components of the amplitudes of the electric and magnetic fields of the wave, (see the expression (2)) we can obtain by use of Maxwell's equations:

$$\begin{aligned} &E_{my} = \alpha_m E_{mx}, \quad H_{mx} = \delta_m E_{mx}, \quad H_{my} = f_m E_{mx}, \\ &\alpha_{1,2} = \mp i, \quad \alpha_{3,4} = \pm iu, \quad \delta_{1,2} = in, \quad \delta_{3,4} = -in, \\ &f_{1,2} = \pm n, \quad f_{3,4} = \pm nu, \quad u = \sqrt{\frac{x_1}{x_2}}, \quad n = \sqrt{\varepsilon_{is}}. \end{aligned} \tag{35}$$

In (35) the quantities, proportional to $x_{1,2}$ of first and higher powers are not taken into account. Therefore the presented below expressions for field have the same inaccuracy. In the phases we will keep a higher accuracy, because the more accurate expressions (31), (31a) for K_m will be used.

Let assume for simplicity, that the dielectric permittivity ε of isotropic homogeneous medium bordering with CLC is equal to ε_{is}. Because of the condition $\varepsilon = \varepsilon_{is}$ and pointed out above accuracy for the amplitudes, the Fresnel's reflections may not be revealed. For the E_{mx} and for the components of amplitude

($\vec{E}_r$) of the reflected wave we obtain (using relations (35) and the boundary conditions for the tangential components of the fields):

$$E_{1x} = (E_x + iE_y)/2,\ E_{2x} = 0,\ E_{3x} = \frac{2}{1+u}\frac{E_x - iE_y}{2},\ E_{4x} = 0,$$
$$E_{rx} = -\left(1 - \frac{2}{1+u}\right)\frac{E_x - iE_y}{2},\ \mathrm{E_{ry}} = -i\left(1 - \frac{2}{1+u}\right)\frac{E_x - iE_y}{2}. \quad (36)$$

Here E_x, E_y -are the components of the incident wave field amplitude.

As it follows from (36) (and from impossibility of revealing of Fresnel's reflections), the reflection has a diffraction character and is selective to the polarization.

Let consider now the isotropic point, in which the helical structure of medium is stipulated only by anisotropy of absorption.

For the reflected wave we will have:

$$E_{rx} = -\left(1 - \frac{2}{1+\sqrt{\varepsilon_1''/\varepsilon_2''}}\right)\frac{E_x - iE_y}{2}, \quad (37)$$
$$\mathrm{E_{ry}} = -\mathrm{i}\left(1 - \frac{2}{1+\sqrt{\varepsilon_1''/\varepsilon_2''}}\right)\frac{\mathrm{E_x - iE_y}}{2}.$$

When $\varepsilon_1'' \ll \varepsilon_2''$ or $\varepsilon_2'' \ll \varepsilon_1''$ (these cases correspond to the greatest values of anisotropy $|\varepsilon_1'' - \varepsilon_2''|/(\varepsilon_1'' + \varepsilon'')$), the total reflection takes place if $E_x = -iE_y$ (diffracting polarization), and reflection is absent if $E_x = iE_y$. When $\varepsilon_1'' = \varepsilon_2''$ (the absence of anisotropy) the reflection is absent in any case.

When $\varepsilon_1'' \to 0$ or $\varepsilon_2'' \to 0$ and, simultaneously, the incident wave has a diffracting polarization, the electric field everywhere in CLC is directed along the x′ axis (or, correspondly, along the y' axis); the diffracting wave propagates in medium without decreasing, because $K_{3,4} \to 0$.

3.3 Irreversibility of circular dichroism

According to the formula (22), irreversibility of plane polarization rotation occurs, when medium is placed in the external magnetic field:

$$|\vartheta_{forw}| \neq |\vartheta_{backw}|.$$

The absorption is one of causes of circular dichroism. The latter determines by imaginary part of ϑ [1,20].

Let consider absorbing CLC in the external magnetic field. When, if ε_1'' and ε_2'' are the quantities of the same order, and $|\varepsilon_1'' - \varepsilon_2''| \ll |\varepsilon_1' - \varepsilon_2'|$. $|g| \ll \varepsilon_1'' + \varepsilon_2''$, from (19), (20) we obtain:

$$|\vartheta_{forw}''| - |\vartheta_{backw}''| = \frac{\omega^2 agB''}{c^2 B'^2}. \tag{38}$$

Here B' = Re B, B$''$ = Im B = Im B and B is determined by formula (16). The relation (38) takes place in nonaccuracy of order $g/(\varepsilon_1'' + \varepsilon_2'')$ outside the RDR.

According to (38), the irreversibility of circular dichroism occurs: $|\vartheta_{forw}''| \neq |\vartheta_{backw}''|$. The circular dichroism irreversibility, as well as all wave irreversibility effects occurs because of simultaneously existence of spatial structure asymmetry of medium ($a \neq 0$) and magnetooptical activity ($g \neq 0$).

4. DISCUSSION

The helical structure under the conditions of frequency dispersion of the dielectric permittivity tensor components leads to a number of medium properties which are discussed above. These properties can be interesting for further researches in the sense described below.

Let us enumerate first some properties which can be of practical importance:

4.1

The properties of CLC near the isotropic point make possible to get narrow RDR, the width and location of which on the frequency axis can be controlled by varying the helix pitch. By varying of the helix pitch we obtain the shifting along the frequency axis RDR with a varying width. The required change of the helix pitch can be realized by superposition of external electric or magnetic fields, and also by the change of temperature.

Ensuring in the desirable frequency interval the existence of the absorption line one can create the region of diffraction reflection. For that one has to ensure the intersection of the depicted in Figure 2 lines in such a way as it is described in the text with respect to Figure 2. Absorption lines can be created by the introduction of absorbing impurities[21].

The waves irreversibility effects make it possible to realize the optical elements having asymmetry. In such elements the reflectivity from a layer, the coefficient of transmission through the layer, the absolute value of polarization plane rotation are not the same for mutually opposite directions of the light transmission (for nonpolarized or plane-polarized light; the incident light is considered to be polarized for the case of rotation of the polarization plane) through the CLC layer. The non-invariance of the dispersion equation under the substitution $K \rightarrow -K$ provides also the non-identity of absorption and circular dichroism for mutually opposite directions of the light transmission.

4.2

Let, in the aspect of theoretical interest, note the same irreversibility of waves. In naturally gyrotropic media the irreversibility takes place due to combination of the magneto-optical activity with the space dispersion. In CLC the irreversibility takes place due to combination the magneto-optical activity with the right hand-left hand asymmetry of the supermolecular structure of CLC. Such a structure also leads to the rotation of the polarization plane, but is not reduced to the gyrotropy. Thus, the irreversibility in CLC by its origin differs from the irreversibility in homogeneous gyrotropic media.

The combination of irreversibility with the periodic nonhomogeneity is an unique situation. It takes place in CLC in the presence of an external magnetic field and is connected with certain changes in the geometry of diffraction reflection.

Further investigations of such a media as well as the helical media with inhomogenity caused by absorption anisotropy are of theoretical interest. Note that the absorption plays a double role in such media : the formation of diffraction reflection and energy dissipation. Because such a double role it is possible, for example, a decrease of absorbed energy with the increase of layer thickness.[22]

4.3

The third aspect is the possibility of reproduction of optical properties of CLC in other regions of wavelengths.

Thus, the considered above properties of irreversibility of CLC one can reproduce in the region of ultrahigh frequencies by means of artificial helical structures. Such structures are considered, for instance, in[23–25]. The media considered in[25] consist of the small size ellipsoids (in comparison with the wave length propagating in a medium). We shall mean the limiting case of ellipsoids, that is to say, small cylinders of the length being much more than the radius is. The cylinders are distributed in the space in such a way as the prolonged molecules in CLC, i.e. they form a helical structure. By superimposing of an uniform magnetic field along the helix axis we get the medium with magnetooptical activity. Then the possibility appears to reproduce the irreversibility effects considered in CLC.

Then owing to the ferromagnetic resonance[26], we shall have the great values of components of the magnetic permeability tensor μ_{ij}, including those of the parameter g_m (the latter is the nondiagonal component of μ_{ij} in the local system of coordinates and causes a gyromagnetic magnetooptical activity); for the great effects of irreversibility there have to be just great values of g_m.

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